Exercise 5F: Solutions

1 Since Jane and Jenny are to be included we still have to select 2 students from the remaining 18. This can be done in ${}^{18}C_2 = 153$ ways.

2 Since the subset already contains the number 5, there are still 4 numbers to be selected from the remaining 9. This can be done in ${}^{9}C_4 = 126$ ways.

3 Since the hand already contains the jack, queen and king of hearts, there are still 2 cards to be selected from the remaining 49. This can be done in ${}^{49}C_2 = 1176$ ways.

4 There are ${}^{10}C_6$ ways of selecting 6 students from 10. We then subtract the number of combinations that include both Rachel and Nethra. If Rachel and Nethra are on the team then we must select 4 more students from the 8 that remain in ${}^{8}C_4$ ways. Therefore, the required answer is

$${}^{10}C_6 - {}^{8}C_4 = 140.$$

5 a 7 students are to be selected from a total of 13. This can be done in ${}^{13}C_7 = 1716$ ways.

b 4 girls are selected from 8 and 3 boys are selected from 5 applying the Multiplication Principle gives ${}^{8}C_4 \cdot {}^{5}C_3 = 700$

c There can be either 4 girls and 3 boys or 3 girls and 4 boys. Applying both the Addition and Multiplication Principles, this can be done in ${}^{8}C_4 \cdot {}^{5}C_3 + {}^{8}C_3 \cdot {}^{5}C_4 = 980$ ways.

d We first consider the number of ways of selecting teams with fewer than 2 boys. There can be either 7 girls and 0 boys or 6 girls and 1 boy. This can be done in ${}^{8}C_7 \cdot {}^{5}C_0 + {}^{8}C_6 \cdot {}^{5}C_1 = 148$ ways. Therefore, the number of teams with at least two boys will be $1716 - 148 = 1568$.

6 a 4 students are to be selected from 10 for the first committee and 3 are to be selected from 10 for the second committee. This can be done in ${}^{10}C_4 \cdot {}^{10}C_3 = 25200$ ways.

b First choose 4 students from 10 for the first committee. This can be done in ${}^{10}C_4$ ways. There are now 6 students left, from which we select 3 for the second committee. This can be done in ${}^{6}C_3$ ways. This gives a total of ${}^{10}C_4 \cdot {}^{6}C_3 = 4200$ different selections.

7 a 7 students are to be selected from 18 for the basketball team and 8 are to be selected from 18 for the netball team. This can be done in ${}^{18}C_7 \cdot {}^{18}C_8 = 1392554592$ ways.

b First choose 7 students from 18 for the basketball team. There are now 11 students left, from which we select 8 for the netball team. This gives a total of ${}^{18}C_7 \cdot {}^{11}C_8 = 5250960$ different selections.

8 a 5 senators are to be selected from a total of 20. This can be done in ${}^{20}C_5 = 15504$ ways.

b There can be either 2 Labor and 3 Liberal senators or 3 Labor and 2 Liberal senators. As there are equal numbers of both types, the number of ways these can be selected is $2 \cdot {}^{10}C_3 \cdot {}^{10}C_2 = 10800$.

c The total number of unrestricted selections is ${}^{20}C_5 = 15504$. We now consider the number of ways of selecting no Labor senator. There are ${}^{10}C_5 = 252$ ways of selecting 5 Liberal senators out of 10. Therefore, there will be $15504 - 252 = 15252$ selections.

9 a There are ${}^{7}C_5 = 21$ ways of selecting 5 numbers out of 7.

b As the sets already contain the numbers 2 and 3, there are still 3 numbers to be chosen from the 5 numbers that remain. This can be done in ${}^{5}C_3 = 10$ ways.

c We subtract the number of subsets that contain both numbers from the total number of subsets. This gives $21 - 10 = 11$ subsets.

10 We select 2 of 5 vowels and 2 of 21 consonants. This can be done in ${}^{5}C_2 \cdot {}^{21}C_2 = 2100$ ways.

11a 4 hearts are to be selected from 13 and 3 spades are to be selected from 13. Using the Multiplication Principle, this can be done in ${}^{13}C_4 \cdot {}^{13}C_3 = 204490$ different ways.

b 2 hearts are to be selected from 13 and 3 spades are to be selected from 13. The remaining 2 cards are to be selected by amongst the 26 cards that are neither diamonds nor clubs. Using the Multiplication Principle, this can be done in ${}^{13}C_2 \cdot {}^{13}C_3 \cdot {}^{26}C_2 = 7250100$ ways.

12a 3 doctors are to be selected from 4 and 1 dentist is to be selected from 4. The remaining position is to be filled with 1 of 3 physiotherapists. Using the Multiplication Principle, this can be done in ${}^{4}C_3 \times 4 \times 3 = 48$ ways.

b 2 doctors are to be selected from 4. The remaining 3 positions are to be chosen from among the $4 + 3 = 7$ non-doctors. Using the Multiplication Principle, this can be done in ${}^{4}C_2 \cdot {}^{7}C_3 = 210$ ways.

13 The girls can be selected in ${}^{4}C_2$ ways. The boys can be selected in ${}^{5}C_2 =$ ways. The children can then be arranged in $4!$ ways. Using the Multiplication Principle gives a total of ${}^{4}C_2 \cdot {}^{5}C_2 \cdot 4! = 6 \cdot 10 \cdot 24 = 1440$ arrangements.

14 The women can be selected in ${}^{6}C_2$ ways. The men can be selected in ${}^{5}C_2 =$ ways. The four people can fill the positions in $4!$ ways. Using the Multiplication Principle gives a total of ${}^{6}C_2 \cdot {}^{5}C_2 \cdot 4! = 3600$ arrangements.

15 There are 4 vowels and 6 consonants. The vowels can be chosen in ${}^{4}C_2$ ways and the consonants can be chosen in ${}^{6}C_3$ ways. The 5 letters can then be arranged in $5!$ ways. Using the Multiplication Principle gives a total of ${}^{4}C_2 \cdot {}^{6}C_3 \cdot 5! = 14400$ arrangements.

16 Each rectangle is defined by a choice of 2 out of 6 vertical lines and 2 out of 5 horizontal lines. This gives a total of ${}^{6}C_2 \cdot {}^{5}C_2 = 150.$

17 There are 13 choices of rank for the first card. 4 cards have this rank, from which we choose 3. There are 12 choices of rank for the second card. 4 cards have this rank, from which 2 will be chosen. This gives a total of $13 \times {}^{4}C_3 \times 12 \times {}^{4}C_2 = 3744$ hands.